



Article

Mathematical modeling of the spraying process of working solution droplets on controlled spray nozzles

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Abstract: In the article, the size of a droplet during garden cultivation determines its initial velocity, the movement of droplets and their trajectory, the rate of ejection, precipitation, evaporation, and flight under the influence of wind, leaf surface moisture, and solution retention in the leaf. the movement of working solution droplets in a sprayer controlled by airflow, equipped with centrifugal spray nozzles, is mathematically modeled. The velocity field of the air flow is represented by a semi-empirical solution based on Prandtl's theory of turbulence, while the movement of droplets is represented by a system of kinematic equations accounting for the Stokes resistance coefficient. The equations are numerically integrated using the Euler correction method (second-order accuracy).

Keyword: sprayer; working solution; droplet kinematics; turbulent air flow; mathematical model; Euler's method; coating homogeneity

Introduction

In the cultivation of agricultural crops, special attention is paid to the technological process of spraying to effectively protect plants and fruit trees from pests, diseases, and weeds [1]. Every year, hundreds of thousands of tons of pesticide working solutions turn into droplets. The rate of precipitation and evaporation, the degree of inertial precipitation, wind blowing and dispersion in the near-surface layer of the atmosphere, moisture on various surfaces of plant cover tissues and the retention of working fluid in them, and the rate of penetration of pesticides into leaf tissue these and other properties of droplets are determined by their size [2, p. 223]. The narrow spectrum of droplet sizes, optimal for each specific case, characterizes a promising technology implemented by the spraying method that meets the constantly growing requirements of resource conservation, high productivity, and environmental friendliness [2, p. 223]. Spray are directed to plants. As a result of the airflow's impact on the plants, the penetration of droplets into the undersides of the leaves increases significantly. The process occurs at significant values of the Reynolds number. Therefore, the movement of the airflow, as well as the movement of droplets, is purely turbulent in nature [4, pp. 43–48]. Turbulence refers to a violent, chaotic, turbulent flow, and it is usually a phenomenon consisting of the spontaneous generation of many nonlinear fractal waves and simple, linear waves of various shapes and sizes without external, random, medium-exciting forces and with their participation when the flow velocity of a liquid or gas in the medium increases. Various models of turbulence have been developed to calculate such flows. They often appear at the boundary, near the wall, and when a wave breaks or overturns. They can form in various currents. In an experiment on turbulence, the exhaust of a steam stream exiting an electric kettle.

Materials and Methods

Let's consider the movement of working fluid droplets using a suspension sprayer as an example. When considering the movement of droplets, we imagine that the droplets exit the nozzles of the sprayer (Figure 1). The sprayer tips are perpendicular to the ground in each direction and at an angle corresponding to the top of the tree, with centrifugal sprayer tips for dispersing droplets located at their outlet. Such a design of the sprayer increases the coverage area of the chemical solution. As

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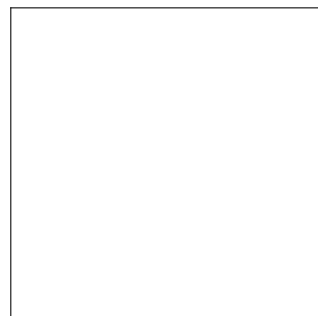
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can be seen, the uniformity of such a sprayer coating depends on the sprayer tips covering the tree with the liquid exiting from it. Therefore, finding the optimal values of these parameters is a task that ensures the homogeneity of the field coating with the toxic chemical.

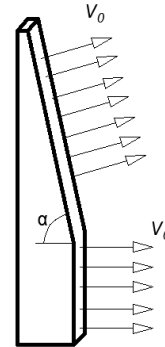


Figure 1. Movement diagram of droplets in the osmosis sprayer.

As stated above, it is necessary to consider the kinematics of droplet flows based on the theory of turbulence. When modeling droplet kinematics, we assume that the mass of the liquid phase per unit volume of air is much smaller than the mass of the air flow. This assumption helps to significantly simplify the problem. Because in this case, the influence of the liquid phase on the dynamics of the air flow can be neglected. Consequently, the problem is divided into two independent problems: the first is modeling and solving the dynamics of the turbulent air flow; the second is modeling and numerically solving the movement of droplets based on the obtained picture of the air velocity field.

Results

Modern high-performance turbulence models can be used to more accurately model the dynamics of turbulent air flow. If high accuracy of air flow velocities is not required, Prandtl's theory of turbulence can be used [5, p. 5].

$$v_z = \frac{14.23 V_0}{z + z_0} \cdot \frac{1}{\left[1 + \left(8.22 \frac{r}{z + z_0} \right)^2 \right]^2},$$

$$v_r = \frac{7.1 V_0 r}{(z + z_0)^2} \cdot \frac{1 - \left(8.22 \frac{r}{z + z_0} \right)^2}{\left[1 + \left(8.22 \frac{r}{z + z_0} \right)^2 \right]^2}, \quad (1)$$

where V_0 is the air flow velocity from the nozzle, m/s;

$z_0 = 4.16R_0$ is called the nozzle pole, m;

z is the longitudinal coordinate, m;

b – slit width;

r is the radii of the spray nozzles, m;

v_z – axial flow velocity, m/s;

v_r – radial flow velocity, m/s.

This solution is valid at a distance from the nozzle, where the ratio of this distance to the nozzle diameter is significantly greater than one. However, it is very suitable for determining the trajectories of droplets, since the process of droplets of the chemicals of interest to plants occurs mainly at the same distances. Air flow velocities working fluid allows you to find the trajectory of droplets. The problem is three-dimensional because gravity acts on the droplets. As a result, the symmetry of the movement of the droplets relative to the axis is not maintained. Therefore, let us consider the movement of droplets emerging from the spray nozzles, which are positioned vertically along the

width of the slit. The trajectories of the droplets obtained from these spray nozzles provide us with a view of the zone covering the surface being treated with the sprayer. This is because the trajectory of the droplets from the lower sprayer gives us the nearest point, while the trajectory of the droplets from the upper sprayer gives us the farthest spray point. For mathematical modeling of such droplets, the kinematic

$$\begin{cases} \frac{dv_{dz}}{dt} = k(v_z - v_{dz}), \\ \frac{dv_{dr}}{dt} = k(v_r - v_{dr}) - g, \\ \frac{dz_d}{dt} = v_{dz}, \\ \frac{dr_d}{dt} = v_{dr}. \end{cases} \quad (2)$$

where v_{dz} , v_{dr} are the axial and radial velocities of the drop, m/s, respectively;

z_d – longitudinal coordinate of the drop, m;

r_d – radial coordinate of the drop, m;

k is the interaction coefficient between the droplet and the air environment;

g is the acceleration of free fall, m/s².

In the nozzle section, the upper part $r = b$ corresponds to the position of the nozzle tip, and the lower part $r = -b$ corresponds to the position of the nozzle tip.

The interaction coefficient between the phases is determined by Stokes' formula [6, p. 427].

$$k = \frac{18\mu}{\rho_0\delta^2}, \quad (3)$$

where ρ_0 is the density of the dust particle material, kg/m³;

δ – effective particle diameter, m;

μ – dynamic air viscosity.

As can be seen from formula (3), the interaction coefficient is strongly dependent on the effective diameter of the drop. The smaller the size of the droplet, the more it is attracted to the airflow. That is, the difference between its velocity and the velocity of the air flow will be small. Thus, the system of expressions (1) and equations (2) is a mathematical model of the movement of droplets.

$$\begin{aligned} \frac{dv_{dz}}{dt} &= k(v_z - v_{dz}), \\ \frac{dv_{dr}}{dt} &= k(v_r - v_{dr}) - g, \\ \frac{dz}{dt} &= v_{dz}, \\ \frac{dr}{dt} &= v_{dr}, \\ v_z &= \frac{14.23 V_0}{z + z_0} \cdot \frac{1}{\left[1 + \left(8.22 \frac{r}{z + z_0}\right)^2\right]^2}, \\ v_r &= \frac{7.1 V_0 r}{(z + z_0)^2} \cdot \frac{1 - \left(8.22 \frac{r}{z + z_0}\right)^2}{\left[1 + \left(8.22 \frac{r}{z + z_0}\right)^2\right]^2}, \\ k &= \frac{18\mu}{\rho_0\delta^2}. \end{aligned} \quad (4)$$

The posed problem cannot be solved analytically. Therefore, Euler's numerical method with recalculation was used to integrate the obtained equations [7, p. 107]. Let us illustrate the principle of this numerical method using an example.

$$\begin{aligned}\frac{dU}{dt} &= k[U_0(x) - U], \\ \frac{dx}{dt} &= U.\end{aligned}\quad (5)$$

(5) we write the finite difference form of the Euler method for the system

$$\begin{aligned}\frac{\tilde{U} - U^n}{0.5 \Delta t} &= k[U_0(x^n) - U^n], \\ \frac{\tilde{x} - x^n}{0.5 \Delta t} &= U^n, \\ \frac{U^{n+1} - U^n}{\Delta t} &= k[U_0(\tilde{x}) - \tilde{U}], \\ \frac{x^{n+1} - x^n}{\Delta t} &= \tilde{U}.\end{aligned}\quad (6)$$

In the specified finite difference scheme, values with index n correspond to values at time $t = n\Delta t$. Thus, the integration of equations (5) occurs at the intermediate values on the right sides. Consequently, the numerical calculation scheme will have a second-order accuracy, i.e., the calculation error will be of the order $o(\Delta t^2)$. The results of the numerical solution of the problem are shown in Figures 2 and 1.3.

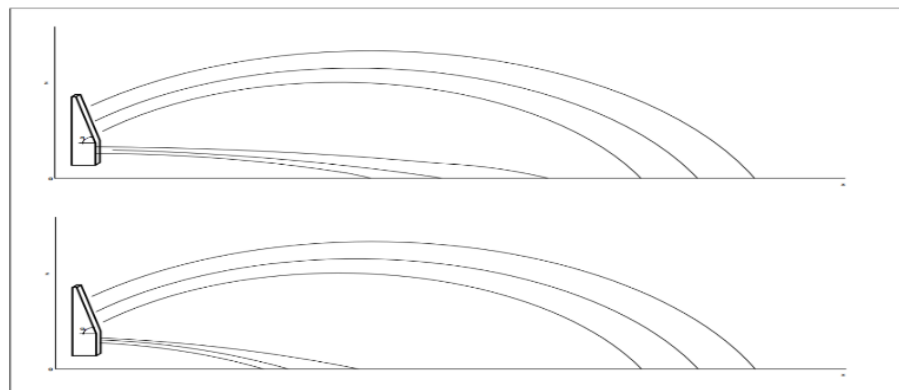


Figure 2. Trajectories of droplets with diameters: (a) 300 μm and (b) 500 μm .

Discussion

These images show the trajectories of droplets coming out of the sprayer with the airflow. Here, the length dimensions are given relative to the nozzle aperture. The sprayer has nozzle openings at two different angles. One slit is horizontal, and the other is at an angle to the horizon. Figure 2 shows the trajectories of droplets with diameters of a) 300 μm and b) 500 μm , located at an angle of 45° to the horizon of the second slit. As can be seen from these figures, the droplets do not cover the falling surface evenly. That is, there is a zone where droplets of chemical pesticides do not fall. Therefore, the direction angle of the second (side) nozzle was adjusted to optimize the surface finish. Figure 1.3 shows the trajectories of droplets with diameters of a) 300 μm and b) 500 μm , where the angle of the second nozzle is 30° . The results obtained show that when the second slit is located at an angle of 30° to the horizon, the toxic chemical

Conclusions

Mathematical modeling of the movement of working solution droplets in a suspension fan sprayer was performed. The model includes a semi-empirical velocity field of the air flow based on Prandtl's turbulence theory and a system of kinematic equations accounting for the interaction

coefficient between the drop and the air (Stokes' formula). The system of equations was numerically integrated using the Euler correction method (a second-order accuracy scheme) and the trajectories of droplets with diameters of 300 and 500 μm were calculated. Calculations were performed at a nozzle outlet air velocity of 50 m/s and a nozzle radius of 0.1 m. When the second nozzle is installed at an angle of 45° to the horizon, the surface of the falling droplets is covered unevenly, forming a zone where no droplets fall. When the angle is reduced to 30° , the homogeneity of the surface coating with the working solution is significantly improved. Thus, the angle of the second nozzle is one of the main parameters directly affecting the coating quality.

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